Simultaneous CO_2 Permit Trading and Output Decisions in a Dynamic Setting with Multiple Sources of Uncertainty

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Abstract

We investigate the simultaneous optimal decisions regarding the production, sales, and inventory of output, as well as the CO_2 permit trading of a regulated firm under the EU ETS. By specifically modeling the uncertainty related to the prices of output and CO_2 permits, the demand for output, as well as the availability of tradable pollution permits, we are able to identify how permit and production decisions influence each other.
The simulations presented here suggest that under the EU ETS, a regulated firm has incentives to decrease pollution to the detriment of production. This feature is even stronger under conditions of higher uncertainty regarding the availability of permits on the market. The impact on total profits is however insignificant, since firms manage to balance sales decreases with inflows from permit trading. Moreover, we find that compared to the case of full availability of permits, firms are inclined to hold extra pollution allowances under uncertainty, in order to avoid incurring large penalty costs at the end of the compliance period. Interestingly, under the EU ETS firms choose to hold higher inventories of output in order to have anticipated information regarding the total realized emissions.
We put forth that possible interventions on the carbon markets should target the variability of the allowance prices. Both very low and high levels of price volatility offer adverse incentives for firms regarding their emission reductions; instead policy makers should direct their actions towards establishing moderate levels of volatility. Additionally, a carbon market where large negative jumps are frequently present cannot constitute an effective pollution reduction policy.

Keywords: *Climate change, EU ETS, Uncertainty, Optimal output level, Dynamic permit trading, Compliance*

1 Introduction

During the past years, it has become increasingly evident that leading a sustainable and less carbon-intensive way of living can no longer be only a choice of the few, but a requirement for the many. A solid consensus on the unfolding of climate change has been reached and it powered the motor for environmental policy changes of many states. The European Union is enforcing the emissions' reduction requirements for its area via a market-based mechanism, called the EU ETS, which came into operation in 2005. Companies belonging to the most polluting sectors of the European industry are assigned ceilings for emissions, reflected in the number of pollution allowances they receive each compliance period. Uncovered emissions are subject to penalties, while unused permits are eligible for sale on different carbon markets. The scheme covered around 10,600 installations initially, with other firms joining later as pollution regulation became binding for additional countries[1] and sectors [EUDirective, 2003]. For a detailed description of the cap-and-trade mechanism and the market characteristics, we kindly refer the reader to Alberola et al. [2008] and Kruger and Pizer [2004], and the references therein.

The introduction of pollution regulation brought along inevitable changes to the business environment of the targeted companies. In this new setting, firms that were active only on the output market, became exposed also to a second, artificially created, market of carbon permits. Debates regarding the efficiency and effectiveness of the EU ETS have been heating up ever since the publication of the European Commission's Green Paper in 2000 [Smale et al., 2006]. According to data provided by Eurostat, the evolution of industrial production after the opening of the EU ETS has been ex post found to be divergent and sector-specific [Alberola et al., 2008]. Smale et al. [2006] investigate the impact of emissions trading on the competitiveness of the European industry, and find that most participating sectors have benefited in general in terms of changes in output, market share, and firm profits. Sijm et al. [2006] document substantial windfall profits for power companies during the first phase of the EU ETS (2005 - 2007), as a result of the CO_2 cost pass-through.

Despite the fact that the carbon market has been extensively analyzed in literature, only little research has been devoted to understanding companies' incentives of balancing business and pollution compliance decisions. Dobos [2007], Wahab et al. [2011], Chaabane et al. [2012], and Li and Gu [2012] adopt deterministic theoretical approaches to investigate the impact of environmental policies on firm's production-inventory strategies. The goal of our work is to study the joint optimal output and permit trading strategy for maximizing total firm value and reaching compliance with assigned emission levels. To the best of our knowledge, this is the first paper to introduce a theoretical model that accounts for simultaneous output production and permit trading decisions, in a context of multiple sources of uncertainty in a dynamic setting.

We are interested in verifying the existence of a *substitution effect* among the two revenue/cost generating sources. Such effects have been empirically documented by Considine and Larson [2006] for the SO_2 market in the US. We hereby extend their work to the EU ETS environment and develop a theoretical model that verifies companies' incentives to coordinate production and emission compliance decisions. In our approach, unsold output and permits are stored from one period to another. We show that these two different types of inventories play a hedging role in case of anticipated shocks in demand or in prices on the two markets.

Moreover, our work is related to the one of Li and Gu [2012], who investigate the influence of emission trading on the production-inventory tactics of regulated companies under the EU ETS. Their study reveals that the possibility of emission trading with banking has a leveling effect on the quantity produced by the firm and causes the output inventory to be larger than in the absence of emission trading. Compared to their work, we introduce various sources of uncertainty. Additionally, we allow for a multi-compliance period decision horizon, mimicking the different phases of the EU ETS, and for the enforcement of penalties on uncovered emissions.

[1] Romania and Bulgaria joined the European Union in 2007.

We specifically model the uncertainty related to the prices of output and CO_2 permits, the demand for output, as well as the availability of tradable pollution permits. The manager's problem is to choose at each moment of time the optimal quantities of output to be produced, stored and sold, and of carbon allowances to be traded that maximize total firm value. We allow for a dynamic setting with multiple compliance periods, in which output decisions impact firm value for the entire optimization horizon, while permit decisions have a temporary, within the compliance period, relevance. We solve the model via a numerical backwards procedure in which the problem is initially solved for the entire planning horizon based on expectations, after which decisions are updated recursively to account for the newly observed information.

Our study reaches three conclusions. Firstly, we find that under the stimulus of a cap-and-trade system, regulated firms tend to slightly decrease pollution to the detriment of production. This feature is even stronger under conditions of higher uncertainty regarding the future availability of carbon permits on the market. Secondly, firm size influences frequency of permit trade. Small firms, with a lower permit demand in absolute terms, tend to wait until the uncertainty regarding their cumulated emissions is solved and only then trade permits. On the other hand, larger firms with a higher permit demand, face an increased probability of low permit availability towards the end of the compliance period that would match their needs. Therefore, they will try to protect themselves against the possibility of incurring penalties by trading permits all along the compliance period, and they will tend to hold extra pollution allowances as long as they do not have complete information regarding their total realized emissions. In this sense, the model captures the empirically observed fact that some firms hold valueless allowances at the end of the compliance periods [Hintermann, 2012], and that larger firms are more active on the market (Weishaar et al. [2012] and Cludius [2012]). Overall, the different strategies of small and large firms result in higher variability of revenues for the former. Lastly, we also confirm the results of Li and Gu [2012] that under EU ETS firms choose to hold higher inventories on average.

The analysis is organized as follows: Section 2 will acquaint the reader with the related literature. Thereafter, we introduce the setting and the assumptions of our model in Section 3, continuing with the presentation of the proposed solution method. Section 4 reveals our main results and verifies their consistency to parameter variations. Finally, in Section 5 we take the opportunity to conclude our thoughts on the implications of our findings for policy implications.

2 Background

Even if tradeble permit markets have been introduced since a couple of decades in the US and only since a few years in the European Union, the literature analyzing pollution allowance markets is increasingly rich and one can already distinguish between different directions of research. Fundamentally, the allowance markets literature is divided into two main branches: one concerned with market design and policy implications, and one focusing on the formation and characteristics of the CO_2 permit price.

Here, in a partial equilibrium setting, we assume a competitive economy in which regulated companies are price takers and optimize their financial situation participating in both output and CO_2 permit markets. Our study is focused primarily on the production decision mechanism of firms that need to comply with emission restrictions. The present paper, therefore, is located at the crossroads between production and environmental economics literature and our micro-level results are expected to be relevant for market policy implications.

Marketable permits have been proposed as a solution for regulating emissions by the pioneering work of Coase [1960] and Dales [1968]. While Coase [1960] highlighted the important role played by the established property rights for mitigating externalities such as green house gas emissions, Dales [1968] was actually the first one to promote the idea of marketable pollution permits. Montgomery [1972] acknowledged the attractiveness of pollution permits for emissions control from an efficiency point of view, by showing that emission trading will ensure reaching compliance at minimum costs.

After the creation in 1990 of the sulfur dioxide allowances market in the United States, the theoretical foundations laid by the previous literature have been extended by studies concerned with the *observed* behavior of players and prices on this engineered market. Cronshaw and Kruse [1996] make use of a discrete time model to show that, in a competitive market for transferable and bankable emission permits, firms opt to allocate emissions across time, such that minimum abatement costs are achieved. Rubin [1996] generalizes the work of Cronshaw and Kruse [1996] to a continuous time setting and derives explicit paths of emissions and permit prices. However, neither of these papers examines the optimality of bankable permits from the point of view of the social planner, which becomes central in the analysis of Rubin and Kling [1993]. Their work relies on a simple optimal control model for investigating the efficiency properties of banking and borrowing, and compares social and individual optimum levels of emissions and production. One of the first studies of pollution markets to analyze the behavior of players under uncertainty is the one of Schennach [2000], which emphasizes the dependence of the present optimal level of emissions not only on current, but also on future abatement costs, electricity demand, and environmental regulations. More precisely, the author reveals how the expected paths of allowance prices and the levels of emissions are updated as new information becomes available on the market. Allowing for uncertainty regarding the realization of output and permits prices, as well as the demand for output, our study departs from the one of Schennach [2000], in that we are explicitly concerned with the output production, storage and sales decisions of regulated firms. In this context, the firm manager operates on two different markets, that of produced output and of marketable permits, and we, therefore, analyze the behavior of firms that act as total profit maximizers, instead of just cost minimizers.

The idea of perceiving the environment as a factor of production has been promoted by Considine and Larson [2006], who performed empirical tests in order to understand firms' decisions of trading and banking SO_2 emission permits. They investigate the substitution possibilities between the environment and other production factors:

> Since output, emissions, future regulations and technology are unknown and penalties are high for emissions that are unmatched with allowances, holding inventories of permits allows firms to address a range of related risks [Considine and Larson, 2006].

We verify the existence and consequences of such substitution effects in our theoretical model.

Recently, with the opening of the European Union's CO_2 trading scheme and the particular characteristics of this young market, researchers started giving considerable attention to the role played by the optimizing behavior of market participants in the formation of carbon permit prices. Fehr and Hinz [2006] design a model in which the players on the carbon market can rely on either long-term irreversible abatement measures or trade permits in the short term to ensure compliance with pollution regulations. Their model reveals that CO_2 permit prices and the compliance probability are influenced by the penalty level and by the initial expected allowance demand. Seifert et al. [2008] build a stochastic equilibrium model in which individual firms take abatement and permit trading decisions, and a central planner minimizes total emission compliance costs. Their main findings reveal that discounted CO_2 prices should exhibit the martingale property and that the modeling of carbon permits should take into account a time and price-dependent volatility structure. Chesney and Taschini [2012] analyze the impact of information asymmetries among profit maximizing regulated firms on carbon price formation. They find that permit price paths depend on the future probability of a shortfall of permits, the penalty paid in case of non compliance with emission limits, and the discount rate.

A series of studies showed interest in the interactions between permit trading and firm profitability. Sijm et al. [2006] analyze the implications of participating in the EU ETS for the power sector, and are interested in understanding how is the internalization of carbon allowance prices reflected into the power price and how does this affect profitability. They document substantial windfall profits for the power sector, due to the cost pass-through of freely allocated allowances. Demailly and Quirion [2006] verify the impact of different allocation approaches on emission trading behavior, firm profitability, production levels and CO_2 leakage. Their application to the cement sector reveals different strengths and weaknesses for grandfathering and output-based allocation schemes. Alberola et al. [2008] lead an empirical study on the relationship between the CO_2 allowance price changes and the economic activity of the sectors included in the scheme. They find that three sectors (combustion, iron, and paper) have a statistically significant impact on carbon price movements.

Finally, our work is related to the one of Li and Gu [2012], who investigate the influence of emission trading on the production-inventory strategy of regulated companies under the EU ETS. By relying on the Arrow-Karlin dynamic production-inventory model, they find that the possibility of emission trading with banking has a leveling effect on the quantity produced by the firm and causes the output inventory to be larger than in the absence of emission trading.

We rely on the findings of the previous literature when building the model and interpreting the results. A comprehensive description of our theoretical model and its assumptions is the main focus of the following section.

3 The Model

3.1 The Firm's Decision Program

The model presented here considers a regulated company operating under the EU ETS and managing one single output line. The company's manager is responsible for formulating the optimal program for both the core-business and the environmental strategies of the firm. The former consists in the decisions regarding production, sales and storage of output, while the latter refers to the optimal CO_2 permit trading schedule. We are interested in analyzing the

implications of these business strategies on the success of the EU ETS and the economy.
The production-sale-storage program will be optimally designed by taking into account pollution regulations. If cumulated emissions exceed the permit endowment at the end of the compliance period, the total firm value is negatively affected by heavy penalty payments. Therefore, rational firm managers will formulate production strategies having in mind the total implied emissions.
We allow for a partial equilibrium setting and assume that both output and permit markets are perfectly competitive and players act as price takers. The firm's instantaneous revenues depend on the quantity of output sold (x_t), the quantity of output produced for future availability (y_t), the corresponding costs with inventory (I_t), and finally the cash flows from trading permits (z_t):

$$\pi_t = P_t{}^Q x_t - C(y_t) - hI_t - P_t{}^{CO_2} z_t \tag{1}$$

where $P_t{}^Q$ and $P_t{}^{CO_2}$ represent the current market output and permit prices, $C(.)$ is the quadratic cost function for production[2], and h refers to unit costs of storage.
We allow the quantity of output sold (x_t) to be bounded from above by the demand level (D_t), but we do not impose the mandatory satisfaction of demand. For example, firms might be in the position to sell less than the quantity demanded in case previous production decisions were based on lower expected demand levels than those realized, or when anticipating high carbon prices. Inventory levels are dynamically modified by the current outflows of sales and the inflows of newly produced output:

$$dI_t = (y_t - x_t)dt \tag{2}$$

We consider a finite decision horizon, consisting in several compliance periods, and spanning over the $[0,T]$ time interval. At the beginning of each period, firms receive for free a number of permits (N) and their compliance status is verified annually. Within a compliance period, the decisions regarding the quantity produced and the permit trading schedule are relevant for the end-of-period pollution compliance status. The profit obtained at the end of each compliance period (τ_T) is non-smooth in nature, due to the possible occurrence of penalties for uncovered emissions:

$$\pi_{\tau_T} = P_{\tau_T}{}^Q x_{\tau_T} - C(y_{\tau_T}) - hI_{\tau_T} - \max\left(0, \Omega\int_{\tau_0}^{\tau_T} y_t dt - \int_{\tau_0}^{\tau_T} z_t dt - N\right)\cdot Pen \tag{3}$$

where $[\tau_0;\tau_T]$ is the time elapsed during a compliance period, and Ω is a conversion factor that helps translating units of output produced into tons of CO_2 emitted.
In this setting, manager's choices need to be taken dynamically, and total firm value will be in focus (TFV), rather than the independent instantaneous profit levels. As we force the decision process to end at time T, we need to account for a *termination payoff* that depends on the state reached. We define the value of the firm in the distant future ($t > T$) as a perpetuity over the profit level (π_T) attained at the end of the active decision-making period:

$$\Pi_T = \int_0^\infty e^{-rt}\pi_t dt = \pi_T \int_0^\infty e^{-rt} dt = \frac{\pi_T}{r} \tag{4}$$

[2]We model the cost as a function of the quantity produced, i.e. $C(y_t) = a_1 y_t + a_2 y_t^2$

The total firm value is given by the accumulated discounted profits:

$$TFV = \int_0^T e^{-rt}\pi_t dt + e^{-rT}\Pi_T \tag{5}$$

where Π_T is the scrap value of the firm after time T, as described above.
To maximize the entire firm value, the manager needs to decide at each moment of time on the quantity of output to sell (x_t), the quantity of output to produce and store for next period (y_t), and the optimal amount of permits to trade (z_t). His information set at each moment comprises the current realizations of prices, output demand, and availability of permits, as well as his updated expectations regarding future periods. The agent's problem can be formulated as follows:

$$\max_{(x_t,y_t,z_t)_{t\in[0,T]}} \mathbb{E}_0\left[TFV\right] \tag{6}$$

We take the chance now to detail the assumptions regarding the stochastic processes followed by the main variables.

3.2 Model Specifications

We allow for a continuous state space, in which carbon and output prices, as well as output demand, are continuous processes that vary stochastically. For the availability of CO_2 permits on the market we assume a discrete stochastic process.
First of all, we model the price of output as a mean-reverting process, trying to capture the empirically documented trend observed in commodity prices. This Gaussian process admits a stationary probability distribution and is attractive from an economic point of view, since it is able to capture the correlated dynamics of prices and demand: when prices reach very high levels, demand tends to reduce and supply tends to increase, producing a counter-balancing effect by pushing prices towards their long-term mean [Schwartz, 1997]. Under the physical probability measure $\mathbb{P}$:

$$dP_t{}^Q = k(\mu_1 - \log P_t{}^Q)P_t{}^Q dt + \sigma_1 P_t{}^Q dW_t \tag{7}$$

where dW_t is the increment of the standard Brownian motion under the $\mathbb{P}$-measure. Applying Itô's lemma to the function $\log P_t{}^Q$ allows us to derive the equivalent dynamics of the return on prices as an Ornstein-Uhlenbeck process:

$$d\log P_t{}^Q = k(a - \log P_t{}^Q)dt + \sigma_1 dW_t \tag{8}$$

where $a = \mu_1 - \frac{\sigma_1{}^2}{2k}$, and $k > 0$ is the magnitude of the speed of reversion to the long-term average log price a. This allows us to solve the equation for the output price [Schwartz, 1997]:

$$P_t{}^Q = e^{e^{-kt}\log P_0{}^Q + a(1-e^{-kt}) + e^{-kt}\frac{\sigma_1}{\sqrt{2k}}W_{e^{2kt}-1}} \tag{9}$$

where $W_{e^{2kt}-1}$ is a time-changed Brownian motion[3]. In expectation, the commodity price takes the value:

$$\mathbb{E}_0[P_t{}^Q] = e^{e^{-kt}\log P_0{}^Q + a(1-e^{-kt}) + \frac{\sigma_1{}^2}{4k}(1-e^{-2kt})} \tag{10}$$

[3] $\int_0^t e^{ku}dW_u = W_{\int_0^t e^{2ku}du} = \frac{1}{\sqrt{2k}}W_{e^{2kt}-1}$, by applying the law: $aW_t = W_{a^2 t}$

Figure 4 in the Appendix illustrates several simulated price paths of output over a period of one year, assuming twenty trading days each month.

Further on, we are interested in modeling the output demand. For coming closer to the empirically observed features of demand, we allow for three characteristics: (*i*) cyclical variations matching economic cycles (expansion and recession), (*ii*) negative correlation with output prices, and (*iii*) a strictly positive lower bound.

$$D_t = max(\alpha_t - \beta {P_t}^Q, Floor) \tag{11}$$

where $\alpha_t := c + (1 + g \cdot t)(1 + \sin(t))$ captures cyclical movements[4], and β represents the constant demand elasticity to prices. $Floor$ is the lower bound for demand. An illustration of a simulated demand path, which follows the business cycle but remains nevertheless stochastic and bounded from below, is presented in Figure 6 in the Appendix.

Daskalakis et al. [2009] investigate the dynamics of the carbon allowances and find that the Geometric Brownian Motion (GMB) with jumps provides the best fit among a series of tested models. The presence of jumps can be explained by the fact that the carbon permit market is known not to have reached full maturity, with variable liquidity and significant sensitivity to policy announcements. We rely on their findings, and allow the CO_2 permit price to vary stochastically and experience jumps of Poisson frequencies and constant magnitude. The price dynamics under the physical distribution $\mathbb{P}$ runs:

$$d{P_t}^{CO_2} = {P_t}^{CO_2}(\mu_2 dt + \sigma_2 dB_t + \phi dM_t) \tag{12}$$

where B_t is a $\mathbb{P}$-Brownian Motion, correlated with W_t ($corr(B_t, M_t) = \rho$), and $M_t = N_t - \lambda_t$ is a compensated Poisson process. Above, N_t is a standard Poisson process[5], marking down the time when the jumps occur, with an average frequency λ per time unit. We allow the jump magnitude to be constant of size ϕ. In between jumps, the price process follows a standard GBM.

The solution to the stochastic differential equation of carbon price is given by:

$${P_t}^{CO_2} = {P_0}^{CO_2} \cdot e^{\mu_2 t} \cdot e^{(-\frac{{\sigma_2}^2}{2}t + \sigma_2 B_t)} \cdot e^{[N_t \log(1+\phi) - \lambda \phi t]} \tag{13}$$

In expectation, the carbon allowance price takes the value:

$$\mathbb{E}_0[{P_t}^{CO_2}] = {P_0}^{CO_2} \cdot e^{\mu_2 t} \tag{14}$$

The parameters used for fitting the price and output dynamics, as well as the discretization procedure needed for simulating demand and prices are fully described in the Appendix.

At the aggregate level, the allowance supply is fixed and given by the cap, and the allowance demand is driven by the level of CO_2 emissions of regulated firms, which result from industrial production activity, energy prices, and weather conditions (Alberola el al., 2009). However, the instantaneous demand and supply of permits noticed on the trading platforms has different driving factors. In order to capture the uncertainty regarding the availability of CO_2 permits to be traded on the market, we model a discrete stochastic process for the supply and demand of allowances, as described in the Appendix C.

[4] A slightly modified function than the one in Li and Gu (2012)

[5] $E[N_t] = \lambda t$, $Var[N_t] = \lambda t$, $\mathbb{P}(N_t = n) = e^{-\lambda t}\frac{(\lambda t)^n}{n!}$

3.3 Solution Method

The manager's problem is to choose the optimal quantities of output and carbon allowances that will maximize total expected firm value (TFV) over the entire decision horizon. Given the initial state variables (the current inventory level of output and permits, as well as the values attained by the stochastic variables), the problem is to find an admissible control law, $\omega = \{[x_0, y_0, z_0], \dots [x_T, y_T, z_T]\}$, that maximizes the profit functional:

$$J_\omega\left(I_0, S_0, D_0, P_0{}^Q, P_0{}^{CO_2}\right) = \mathbb{E}_0\left[e^{-rT}\Pi_T(x_T, y_T, z_T) + \int_0^T e^{-rt}\pi_t\left([I_t, S_t, D_t, P_t{}^Q, P_t{}^{CO_2}]|x_t, y_t, z_t\right)dt\right] \tag{15}$$

subject to the system of equations of motion:

$$\begin{aligned}
dI_t &= (y_t - x_t)dt \\
dS_t &= z_t dt \\
dP_t{}^Q &= k(\mu_1 - \log P_t{}^Q)P_t{}^Q dt + \sigma_1 P_t{}^Q dW_t \\
dP_t{}^{CO_2} &= P_t{}^{CO_2}(\mu_2 dt + \sigma_2 dB_t + \phi_t dM_t)
\end{aligned}$$

We apply now the standard dynamic programming technique, by splitting the decision sequence into two parts, the current period and the entire remaining horizon after that. The optimal firm value (J_t) at time t can be written recursively as:

$$J_t\left(I_t, S_t, D_t, P_t{}^Q, P_t{}^{CO_2}\right) = \max_{x_t, y_t, z_t} \mathbb{E}_t\left[\pi_t(x_t, y_t, z_t) + e^{-r}J_{t+1}(I_{t+1}, S_{t+1}, D_{t+1}, P_{t+1}{}^Q, P_{t+1}{}^{CO_2}|x_t, y_t, z_t)\right] \tag{16}$$

where $S_t = \int_0^t x_u du + N$ defines the current total position in the carbon permits. The firm value at final time T consists in the profit level of the period (π_T) and the scrap value of the firm (Π_T):

$$J_T\left(I_T, S_T, D_T, P_T{}^Q, P_T{}^{CO_2}\right) = \max_{x_T, y_T, z_T} \mathbb{E}_T\left[\pi_T(x_T, y_T, z_T) + \Pi_T(x_T, y_T, z_T)\right] \tag{17}$$

where we consider Π_T as defined in Section 3.1.

Given the expected final firm value, we proceed by solving the optimal control problem backwards, as described in [Bertsekas, 1976]. The numerical procedure starts with discretizing the state and the solution space. Based on the expectations regarding the final profit, we can compute the value of the firm at time ($T-1$), and so on.

We consider an active decision horizon of four consecutive compliance periods in length of one year each, with quarterly production, sales, storage, and permit trading decisions. After this period, profits are expected to remain in perpetuity at the level reached in the final quarter of the active decision horizon. At the end of each compliance period, unused allowances expire and cannot be used in subsequent periods[6]. We assume that the firm has no uncovered emissions from periods prior to the decision horizon. Each year, the company needs to cover all emissions from production within the current year for quarters one to three. Emissions from production in the fourth quarter are covered in the following year. The firm starts with a positive output inventory, and an annually renewable endowment of permits of N. At the end of each compliance

[6] Banking of permits was not allowed during the first phase of EU ETS.

period, a penalty is to be incurred for emissions that are not covered by CO_2 allowances.
The quantity sold (x_t) is naturally bounded from above by the expected minimum between demand and existing inventory. Keeping in mind that demand is by assumption bounded from below, the upper bound for sales computed at time s for time t, with $s < t$, is given by[7]:

$$\mathbb{E}_s\left[min(D_t, I_s)\right] = I_s\left[1 - \mathbb{1}_{Floor \leq I_s} \cdot \mathbb{N}(-q3) - \mathbb{N}(-q_1) \cdot \mathbb{N}(q_3)\right] + Floor \cdot \mathbb{1}_{Floor \leq I_s} \cdot \mathbb{N}(-q_3)$$
$$+\alpha_t \cdot \mathbb{N}(-q_1) \cdot \mathbb{N}(q_3) - \beta e^{\left[e^{-k(t-s)} \log P_0{}^Q + a(1-e^{-k(t-s)}) + \frac{\sigma_1{}^2}{4k}(1-e^{-2k(t-s)})\right]} \cdot \mathbb{N}(-q_2) \cdot \mathbb{N}(q_4)$$

where $\mathbb{N}(.)$ is the cumulative density function of the standard normal distribution[8].
Given the realizations of the stochastic variables, the rational manager defines an optimal *contingency plan*, in which the control variables are chosen based on the knowledge of previously realized shocks and updated expectations regarding the distribution of these variables. The gathering of information plays a fundamental role in the decision process. At each moment of time, the agent will design an expected path for the current and all future remaining periods, but will execute only the optimal policy regarding the present. In the next period, he will observe the new realizations of the variables, and redefine his strategy for the current and all next periods, implementing only the current one, and so on. In total, the agent will solve as many optimization problems as the number of decision time points.
More explicitly, we allow the manager to rely on an *open-loop feedback control scheme*, as defined in Bertsekas [1976], in which decisions are taken recursively: based on the arrival of new information, conditional probabilities are recalculated at each moment of time and new expectations for future realizations are formed and used when solving for the optimal control path. The problem is naively solved at each decision point for the entire remaining path, by finding the control input that would be optimal if all the stochastic variables were replaced by their updated expected values. The probability measures of the exogenous disturbances (demand and prices) are independent of the chosen controls (output quantity and amount of permits traded).

4 Results and Sensitivity

4.1 Scenarios and Indicators

We run our model for a horizon consisting in four compliance periods, each in length of one year. The manager takes quarterly decisions regarding the production, sale, and storage of output, as well as decisions linked with the permit trading strategy of the firm. The scrap value of the firm is considered to be a perpetuity over the profit reached in the final quarter, adjusted by

[7] The full derivation is presented in Appendix A

[8]
$$q_1 = \frac{\log \frac{\alpha_t - I_s}{\beta} - e^{-k(t-s)} \log P_0{}^Q - a(1-e^{-k(t-s)})}{\frac{\sigma_1}{\sqrt{2k}}\sqrt{1-e^{-2k(t-s)}}}; q_2 = q_1 - \frac{\sigma_1}{\sqrt{2k}}\sqrt{1-e^{-2k(t-s)}}$$
$$q_3 = \frac{\log \frac{\alpha_t - Fl}{\beta} - e^{-k(t-s)} \log P_0{}^Q - a(1-e^{-k(t-s)})}{\frac{\sigma_1}{\sqrt{2k}}\sqrt{1-e^{-2k(t-s)}}}; q_4 = q_3 - \frac{\sigma_1}{\sqrt{2k}}\sqrt{1-e^{-2k(t-s)}}$$

the probability of survival of the firm.

The solution is represented by a group of three vectors, each containing a number of elements equal to the number of decisions taken (for each quarter of the four annual compliance periods). Let *nq* denote the number of quarters of active decisions, then the solution to the optimal control problem can be defined as the multidimensional control law $\omega = \{X,Y,Z\}$, with the sales and production decisions strictly positive, i.e. $X,Y \in {\mathbb{R}_+}^{nq}$, and the permit trading decision in the set of integer numbers, i.e. $Z \in \mathbb{Z}^{nq}$.

To be able to comprehend the changes in managerial decisions due to the introduction of the EU ETS regulation, we consider three different scenarios. First of all, we model the optimal decisions taken by the manager in the absence of pollution regulation, when the objective function consists in maximizing net revenues from producing, storing, and selling output. We fix this setting to be the benchmark case (denoted *Model 0*), serving as a comparison base for the results obtained under pollution regulation. Additionally, we evaluate two *focus* cases of a European company regulated by the EU ETS. In the first setting, hereafter *Model 1*, the availability of permits to be traded on the market at each decision point is stochastic for both the supply and the demand side (and modeled as described towards the end of Section 3, Subsection 3.2). In the second case, entitled *Model 2*, the firm does not need to worry about a potentially restrictive availability of permits. Instead, his demand or supply of permits is assumed to be fully met throughout the entire decision horizon.

By solving the optimal control problem for the benchmark and the focus cases, we expect to observe strategical differences among the three scenarios regarding the firm's core business and pollution compliance decisions.

The results we present here are based on one thousand runs of the model, in which we hope to have captured a large part of the different settings, generated by the possible paths of the stochastic variables. In order to ease the interpretation of results, we construct several indicators meant to capture the differences between the results of the benchmark and the two additional models (see Table I). Two groups of indicators were used, differentiating between the results related to core-business activities and those linked to the pollution compliance decisions.

We observe that across the different simulations, the strategy the company chooses every year is very similar and specific to the quarter. We present below only summary statistics, while the results are displayed in full version in the Annex.

Table I: **Indicators of Difference**

Indicator	Formula	Additional Variables
Group 1: Core-business decisions		
Production change	$\Delta y_i\% = \frac{\bar{y}^{M_i} - \bar{y}^{M_0}}{\bar{y}^{M_0}}$	$\bar{y}^{M_i} = \frac{1}{ns} \sum_{s=1}^{ns} \sum_{q=1}^{nq} y_{s,q}{}^{M_i}$
Profits change	$\Delta \pi_i\% = \frac{\bar{\pi}^{M_i} - \bar{\pi}^{M_0}}{\bar{\pi}^{M_0}}$	$\bar{\pi}^{M_i} = \frac{1}{ns} \sum_{s=1}^{ns} \sum_{q=1}^{nq} \pi_{s,q}{}^{M_i}$
Profit volatility	$\Delta \sigma_i\% = \frac{\bar{\sigma}_\pi{}^{M_i} - \bar{\sigma}_\pi{}^{M_0}}{\bar{\sigma}_\pi{}^{M_0}}$	$\bar{\sigma}_\pi = \frac{1}{ns} \sum_{s=1}^{ns} \sqrt{\frac{1}{nq-1} \sum_{1}^{nq-1} (\pi_{q,s} - \bar{\pi}_s)^2}$
		$\bar{\pi}_s = \frac{1}{nq} \sum_{q=1}^{nq} \pi_{q,s}$
Storage excess	$\Delta Sx_i\% = \frac{\bar{Sx}^{M_i} - \bar{Sx}^{M_0}}{\bar{Sx}^{M_0}}$	$\bar{Sx} = \frac{1}{ns} \sum_{s=1}^{ns} \left(\frac{1}{nq} \sum_{q=1}^{nq} \mathbb{1}_{I_{q,s} > x_{q,s}} \right)$
Group 2: Compliance decisions		
Trading ratio	$TR_{q_t} = \frac{\frac{1}{ns} \sum_{s=1}^{ns} x_{q_t,s}}{\frac{1}{ns} \sum_{s=1}^{ns} S_{q_t,s}}$	$x_{q_t} = \frac{1}{ny} \sum_{g=1}^{ny} x_{t,g}$
Emissions' coverage	$EC_{q_t} = \frac{\frac{1}{ns} \sum_{s=1}^{ns} S_{q_t,s}}{\frac{1}{ns} \sum_{s=1}^{ns} \sum_{t=1}^{4} S_{q_t,s}}$	$S_{q_t} = \frac{1}{ny} \sum_{g=1}^{ny} S_{t,g}$
Trade frequency	$TF_i = \frac{1}{ns} \sum_{s=1}^{ns} \frac{\sum_{q=1}^{nq} \mathbb{1}_{x_q \neq 0}}{nq}$	

Note: Indicators in Group 1 are computed as variations in results of the focus models from the benchmark. Indicators in Group 2 will be used for comparison purposes between the trading strategies of the two focus cases. The base case is identified with the index 0, while the two focus cases are marked by the index i, with $i = 1,2$, corresponding to Model 1 and Model 2, respectively.

In the first group, *Production change* ($\Delta y_i\%$) captures the variation in yearly production levels. *Profit change* ($\Delta \pi_i\%$) measures mean differences in yearly total revenues. The *Profit Volatility* indicator ($\Delta \sigma_i\%$) reveals the impact of emissions trading on the yearly variability of quarterly profits as compared to the reference case. *Storage excess* (ΔSx_i) indicates the amount of time the firm decides to increase its output inventory, as percentage in the total number of decision points.

In the second group, the *Trading ratio* (TR_{q_t}) captures the average number of permits traded on the market in a specific quarter across the four years, as percentage of the former position in permits. The *Emissions' coverage* (EC_{q_t}) quantifies how far is the firm each quarter (in terms of total owned allowances) from reaching the annual pollution compliance. We compute also the *Trade frequency* over the entire horizon.

Symbols: x_t - sales of output, y_t - output quantity produced, z_t - number of permits traded, ns - total number of simulations, nq - total number of decision quarters, π_t - firm's profit, D_t - demand of output, S_t - accumulated position of permits, I_t - output inventory, $t = 1,2,3,4$ - quarter index in a year, x_{q_t} - average sales in quarter t over the entire decision horizon, S_{q_t} - permit holdings in quarter t over the entire decision horizon, ny - number of years.

4.2 Results

We are primarily interested in quantifying differences in core-business and pollution compliance strategies that appear due to the introduction of the emission regulation. Tables II and III below illustrate changes in optimal behavior with regard to production and compliance strategies, respectively. We first show the results obtained when calibrating the model to reflect average empirical market characteristics, while in the following sections we present the modifications in results when key parameters take different values.

Table II: **Differences in Core-business Strategies**

Indicator	Notation	Model 1	Model 2	$\|M_1 - M_2\|$
Production change	$\Delta y\%$	-4.57%	-4.12%	0.45%
		(-30.87)	(27.65)	(-3.66)
Profits change	$\Delta\pi\%$	-0.05%	-0.46%	0.41%
		(-0.13)	(1.11)	(0.99)
Profit volatility	$\Delta\sigma\%$	+199.5%	+229.3%	+29.8%
		(69.91)	(-80.21)	(-7.40)
Storage excess	$\Delta Sx\%$	+9.71%	+14.13%	4.42%
		(-5.69)	(16.75)	(11.98)

Note: The table captures the results for the indicators in Group 1, i.e. measures of difference between the solutions obtained for the focus *Models 1* and *2*, in comparison with the base case *Model 0*. All results are averages over one thousand independent runs of the model. T-statistics are in the brackets.

Our simulations show that companies subject to European pollution regulation tend to reduce yearly production by around 4% below the zero-regulation scenario, as a consequence of their decision to target decreases in emissions from the manufacturing of output. When the liquidity of permits on the carbon market is uncertain (*Model 1*), the impact is slightly stronger. Exposed to an additional source of uncertainty, companies operating under *Model 1* are significantly more cautious regarding their total emissions. It appears that the costs imposed by the trading of allowances and the possible penalty payments determine the contraction of emissions, making this result policy-relevant: regulators could impact the size of these financial burdens, by either establishing a certain level of the penalty for uncovered emissions, or by intervening on the carbon markets for giving a boast to allowance prices. The problem is two-sided, since too high compliance costs could have negative consequences on the economy, should they cause a sharp drop in output supply, while too low costs could result in no emission reductions.

With the shrinkage of production, the decrease in total profits comes along. This feature emerges as an implicit consequence of the drop in sales, which represents the main driver of firm profitability. The effect is however not statistically significant for either of the two models, since firms manage to compensate the drop in output sales with an increase in profit from trading allowances.

The exposure to EU pollution regulation brings along increases in the volatility of profits, a direct consequence of the fact that firms are now vulnerable to new sources of uncertainty. Moreover, the conditions of *Model 2* favor an augmented effect on revenue volatility, due to the design of the permit-trading strategy. We notice that *Model 2* opts for a more frequent liquidation of permit stocks than *Model 1* (Liquidation rate indicator, Table III). The results can be further understood by observing the permit strategies of the firms. First of all, firms operating under both models have incentives to increase their entire stock of permits in the first phase of each compliance period and sell the unnecessary allowances in the second part of each year, in order to reach perfect compliance and hold no valueless permits at the end. Some differences appear in the optimal permit-trading strategies chosen under the two settings. Having the certainty of being able to trade as many rights as desired, companies acting under the conditions of *Model 2* start clearing the estimated excess permit positions from the second quarter, making sure that at the end of the compliance period they have exactly the needed number of permits. On the other hand, firms constrained to the setting of *Model 1* are not sure to be able to trade all needed permits at the end of the compliance period, due to the stochasticity inherent in the liquidity of carbon markets. This leads firms to continue accumulating permits in advance, in order to be sure of avoiding penalties at the end of the period. The size of the trade in the intermediary quarters depends on the realizations of permit availability at each moment of time and on their cumulated and forecasted emissions from production. The different permit trading strategies can be observed in Table III, with the help of the *Trading ratio* (TR_{q_t}) indicator, which measures the magnitude of one quarter's trades with respect to the previously-held stock of permits. To bring additional evidence to the different compliance strategies under the two models, one can consider also the *Emissions' Coverage* index, which illustrates the progressive accumulation of permits to reach compliance under both models.

Table III: **Differences in Permit Trading Strategies**

Indicator	Model 1		Model 2	
Trade frequency	65.23%		71.54%	
Liquidation rate	0.57%		17.56%	
Quarter	Trading ratio TR_{q_t}	Emissions' Coverage EC_{q_t}	Trading ratio TR_{q_t}	Emissions' Coverage EC_{q_t}
q_1	+210.73%	92.67%	+164.68%	90.55%
q_2	+16.94%	287.96%	-33.28%	239.67%
q_3	-70.30%	336.74%	-37.47%	159.92%
q_4	0%	100%	0%	100%

Note: The table captures the results for the second group of indicators, i.e. measures concerned with the optimal strategies of permit trading at each decision point and the yearly compliance to regulated emission levels. Additionally, the *Liquidation rate* measures the average amount of times the firm chooses to sell its entire endowment of permits over the decision horizon. The results refer to averages over one thousand independent runs of the model.

The impact of allowance trading on the holding of inventory levels is captured by the *Storage excess* indicator in Table II. We confirm the results of Li and Gu (2012) that, under the EU ETS, firms choose to hold higher inventories on average relative to the level of sales. Companies more frequently store unsold output from one period to the other, as compared to the benchmark case.

We understand this as a consequence of the fact that the production of output is undertaken in early quarters for later periods, since companies are happy to have ahead-of-time information regarding the total cumulated production (and emissions) over the year for updating their pollution compliance strategy and avoiding to incur penalties. In order to avoid holding unnecessary high levels of permit stocks, whose redemption value is zero after the end of the compliance period, firms opt for increasing the level of output inventory that could be used for subsequent sales, either in the current or in the following years. This can be understood as a *substitution effect* between the two types of inventories, whose empirical existence needs to be verified.

4.3 Interventions on the Carbon Market

We relax now the assumptions regarding the parameter values used in our model, and check modifications in behavior caused by different values of those parameters that policy-makers are able to influence. We are particularly interested in possible interventions on the carbon markets and, therefore, look at implications resulting when modifying the parameters responsible for the variation in allowance prices. More precisely, the analysis concerns the volatility of carbon prices (σ_2), as well as the frequency (λ) and the magnitude (ϕ) of jumps.
Figure 1 captures changes in some of the most important indicators (as computed in Table I) regarding the core-business and permit trading strategies of firms.

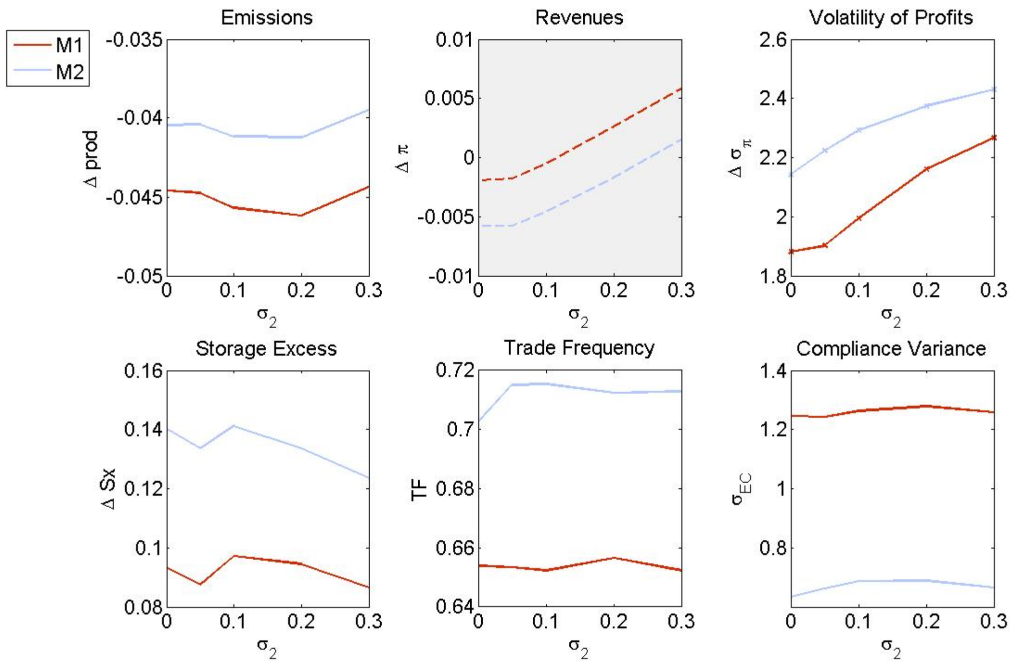

Figure 1: Sensitivity to the Volatility of Carbon Price

First of all, yearly revenues appear to be increasing in the level of volatility. The impact is however shallow; assuming that firms expect high volatility levels, they will readjust their

business strategies, such that firm value is not significantly affected.
On the one side, significant changes do occur in firms' emissions strategies, where impacts seem to be non-linear. Carbon markets with extreme volatility levels (either very low of very high) are less effective pollution-reduction policies than markets with moderate levels of volatility. Firstly, without uncertainty regarding the evolution of carbon prices ($\sigma_2 = 0$), the cap-and-trade market comes close to a pollution tax system. Low volatility limits the upside growth potential of carbon prices, rendering expected compliance costs smaller and, therefore, less powerful in determining emission contraction. At the opposite end, the presence of very high volatility levels renders the forecasting extremely difficult and lowers the firms' propensity of relying on the trading of permits for compensating losses from decreases in sales. Regulators' intervention should focus on the maintenance of a moderate level of volatility on the market.
Another significant effect of increases in carbon price variability consists in the build-up of yearly profit volatility compared to the zero-regulation case. This comes as no surprise, as firms are exposed to an increasingly potent uncertainty source.
Lastly, the results show that changes in the volatility of allowance prices have a constant effect on the other indicators, and we therefore remain at the conclusions reached in the case of the standard parameter set in Section 4.2.

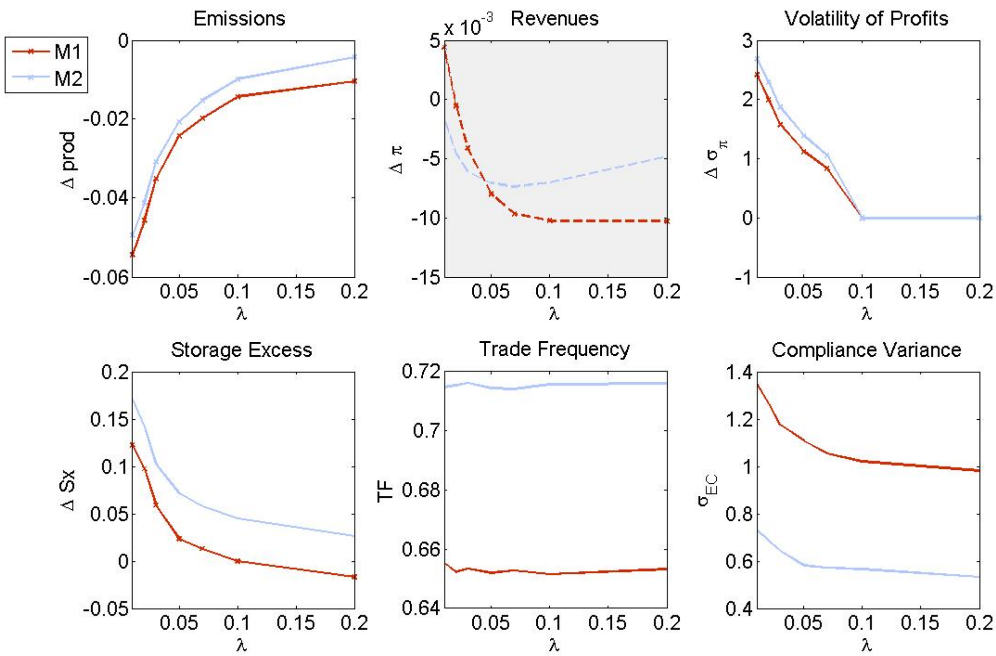

Figure 2: Sensitivity to the Frequency of Jumps in the Carbon Price

The occurrence of jumps in allowance prices has been a reality since the opening of the European carbon markets in 2005. We are now interested in analyzing changes in firm decisions when varying the frequency of jumps. We first allow for jumps of constant and negative magnitude. In a second step we check implications in the case of a large range of jump sizes of both positive and negative signs.

Our results point to the fact that when negative jumps are expected to take place less frequently (low λ levels), firms choose to significantly lower their emissions. Revenues remain close to the levels of the no-regulation scenario, without statistically significant differences between the two models. Moreover, with less frequent decreases in carbon prices, firms experience a higher volatility of revenues and their propensity to carry over output storage is augmented. Figure 2 also captures the constant impact on the frequency of trade and the variability in the compliance strategy.

Overall, the effects caused by varying the presence of negative jumps on the carbon markets carry an important message for regulators. First of all, eliminating negative jumps in allowance prices leads to large reductions in emissions without compromising firm profits. Secondly, carbon markets where prices are expected to experience frequent declines are ineffective in changing firm behavior, such that their output sales and storage strategies, as well as their emissions, will remain close to the zero-regulation scenario. Only after diminishing the frequency of negative jumps can the EU ETS become an effective policy for fighting climate change.

We move on to analyze variations in managerial decisions for different magnitudes of jumps present in the dynamics of the allowance prices (Figure 3). We hold this time the frequency of jumps constant, and check the implications of both positive and negative jumps of various sizes. The changes in indicators experienced when increasing the magnitude of negative jumps confirms our previous observations. The larger the expected drop in carbon prices, the less efficient the market mechanism becomes for changing firm behavior and lowering emissions. In this case, all core-business indicators move towards the levels of the zero-regulation scenario.

However, increases in the size of positive jumps in carbon prices lowers emissions and increases overall firm revenues. With increasing compliance costs, firms are more determined to avoid penalties and accumulate permit stocks, by purchasing large allowance quantities especially in the first part of the compliance period. Over the year, as their information regarding total emissions is increasingly reliable, firms can start selling permits at the now higher price and cash in revenues from permit trading, with a positive effect on total realized profits.

Higher uncertainty due to more intense possible permit price soars leads to increases in profit volatility and in output storage excess, being a signal that, however beneficial for pollution reduction, extreme expected price rallies could prove damaging for firm welfare. These trends are expected to be augmented for risk-averse managers that take into account the minimization of profit volatility together with the maximization of expected profits when designing optimal firm strategies.

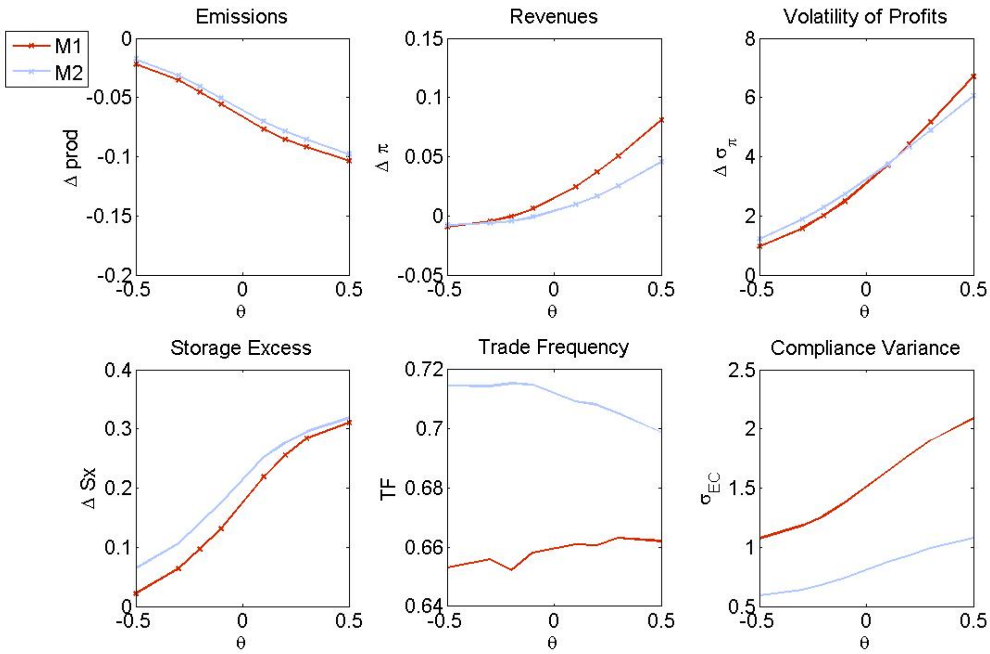

Figure 3: Sensitivity to the Magnitude of Jumps in the Carbon Price

5 Conclusions and Further Research

The purpose of this paper was twofold. Firstly, we explored the joint optimal output and permit trading strategies for maximizing total firm value and reaching compliance with assigned emission levels. Secondly, drawing on the results of pollution and production decisions, we were after pollution policy implications. Our analysis approach consisted in solving a dynamic optimization problem of firm's profits maximization. We solved the model numerically relying on a backwards procedure.

The simulations presented here suggest that the introduction of the EU ETS favored changes in firm behavior at several levels. Incentivized by pollution regulation, companies adjust their core-business strategies and coordinate them with pollution compliance decisions. Our results point that regulated firms have incentives to decrease pollution to the detriment of production. This feature is even stronger under conditions of higher uncertainty regarding the availability of permits on the market. Moreover, we find that compared to the case of full availability of permits, firms are inclined to hold extra pollution allowances under uncertainty, in order to avoid incurring large penalty costs at the end of the compliance period. Finally, under the EU ETS firms choose to hold higher inventories of output in order to have anticipated information regarding the total realized emissions. By allowing for storage flexibilities and manipulation of total production, our theoretical model was built general enough to be relevant also in case EU ETS extends to include additional sectors in the economy.

Our results target both the micro and macro perspectives. On the one hand, managers could benefit from our analysis by verifying whether their strategies are built to be robust enough to different kinds of shocks on the output and carbon markets. Secondly, policy makers could use our model as a measuring tool for how changes in the environmental regulations impact firms' incentives to reduce pollution or modify output supply. We argue that a large weight in the decision process is given by the costs faced in case of non-compliance: too large costs foster a significant contraction of output, while too low penalties render the pollution regulation ineffective. The model is also relevant for the frequency with which firms choose to trade permits, depending on their expectations regarding their realized emissions and future availability of permits. However, we call on future research for the empirical validation of these findings. Further improvements to the model could come from dropping the risk-neutral agent hypothesis, and accounting instead for the variance of returns when taking optimal decisions.

We have seen that interventions on the carbon markets should be targeted at the variability of the allowance prices. Both very low and high levels of price volatility offer adverse effects for firms regarding their emission reductions; instead policy makers should target moderate levels of volatility. Additionally, a carbon market where large negative jumps are frequently present cannot constitute an effective pollution reduction policy.

The carbon market did not clear up in the first phase of the EU ETS, with a number of permits equal to 2.7% of cap expiring unused [Hintermann, 2012]. In the aforementioned paper, the author leads an empirical study in which firms view their emissions as stochastic, due to the inherent uncertainty in the output demand they need to satisfy fully at each moment of time. In absence of full control over their emissions, firms buy extra permits in order to hedge against possible penalties. Our results confirm that firms are better off by holding extra pollution allowances. However, since firms are not constrained to satisfy demand, this decision is motivated in our setting by the uncertainty related to the availability of permits on the market. Moreover, the risk fostered by the random amount of permits existing on the market has been documented to have a significant influence also on the changes in the CO_2 permit price (Chesney and Taschini [2012], Hintermann [2012]). We believe that modifying our model, by accounting for the impact of permit uncertainty on price, could bring further insights into firms' permit trading decisions and their impact on production strategies.

In our model, the markets are perfectly competitive with atomistic firms. If we allow instead for a large player that can afford to inject large sums of money into the carbon market, we expect the firm to try to benefit from the correlation between output and permit prices and to manage his permit strategy such that it results in impacts on the output price. By going long in large quantities on the carbon market, the player can drive the price of permits up, which would result in an increase in output price due to correlation[9]. Of course, the strategy is expected to have its limits, given by the tradeoff between output revenues and pollution costs.

[9]This intuition is confirmed in the results derived in Hintermann [2011].

Appendix

A Expected Upper Bound On Sales

The dynamic programming approach proposes the finding of optimal controls for the period in course, by taking into account the effects of these choices on the forthcoming periods. In this spirit, solutions regarding the future amount of sales (x_t) are undertaken compatibly with the expected demand levels for output (D_t). There is a natural upper bound for future sales given by the minimum between the existing inventory and the future demand level ($x_t \in [0, E_0[\min(I_0, D_t)]]$). This appendix presents the analytical solution to the upper boundary of this interval. We define output demand as a function of business cycle and output price:

$$D_t = max(\alpha_t - \beta P_t^Q, Floor) \tag{18}$$

where:

$$\begin{aligned} \alpha_t =& c + (1+gt)(1+sin(t)) \\ P_t^Q =& exp\left[e^{-kt}\log P_0^Q + a(1-e^{-kt}) + e^{-kt}\frac{\sigma_1}{\sqrt{2k}} W_{e^{2kt}-1}\right] \end{aligned}$$

We start by explicating the expectation:

$$\begin{aligned} \mathbb{E}_0[\min(I_0, D_t)] =& \mathbb{E}_0[I_0 \cdot \mathbb{1}_{D_t > I_0}] + \mathbb{E}_0[D_t \cdot \mathbb{1}_{D_t \leq I_0}] \\ =& I_0 \cdot \mathbb{E}_0[\mathbb{1}_{D_t > I_0}] + \mathbb{E}_0[D_t \cdot \mathbb{1}_{D_t \leq I_0}] \\ =& T_1 + T_2 \end{aligned}$$

where:

$$T_1 = I_0 \cdot \mathbb{E}_0[\mathbb{1}_{D_t > I_0}] \tag{19}$$

and

$$T_2 = \mathbb{E}_0[D_t \cdot \mathbb{1}_{D_t \leq I_0}] \tag{20}$$

We proceed by computing the two terms separately:

$$\begin{aligned} T_1 =& I_0 \cdot \mathbb{E}_0[\mathbb{1}_{D_t > I_0}] \\ =& I_0 \cdot \mathbb{P}(D_t > I_0) \\ =& I_0 \cdot [1 - \mathbb{P}(D_t \leq I_0)] \\ =& I_0 \cdot \left[1 - \mathbb{P}(max(\alpha_t - \beta P_t^Q, Floor) \leq I_0)\right] \\ =& I_0 \cdot \left\{1 - [\mathbb{P}(Floor \leq I_0) \cdot \mathbb{P}(\alpha_t - \beta P_t^Q < Floor) + \mathbb{P}(\alpha_t - \beta P_t^Q \leq I_0) \cdot \mathbb{P}(\alpha_t - \beta P_t^Q \geq Floor)]\right\} \\ =& I_0 \cdot \left\{1 - \left[\mathbb{1}_{Floor \leq I_0} \cdot \mathbb{P}\left(P_t^Q > \frac{\alpha_t - Floor}{\beta}\right) + \mathbb{P}\left(P_t^Q \geq \frac{\alpha_t - I_0}{\beta}\right) \cdot \mathbb{P}\left(P_t^Q \leq \frac{\alpha_t - Floor}{\beta}\right)\right]\right\} \\ =& I_0 \cdot \left\{1 - \left[\mathbb{1}_{Floor \leq I_0} \cdot \left(1 - \mathbb{P}\left(P_t^Q \leq \frac{\alpha_t - Floor}{\beta}\right)\right) + \mathbb{P}\left(P_t^Q \geq \frac{\alpha_t - I_0}{\beta}\right) \cdot \mathbb{P}\left(P_t^Q \leq \frac{\alpha_t - Floor}{\beta}\right)\right]\right\} \end{aligned}$$

We can now solve for $\mathbb{P}(P_t{}^Q \leq \frac{\alpha_t - Floor}{\beta})$:

$$\begin{aligned} P_t{}^Q \leq & \frac{\alpha_t - Floor}{\beta} \\ \Rightarrow exp\left[e^{-kt} logP_0{}^Q + a(1-e^{-kt}) + e^{-kt}\frac{\sigma_1}{\sqrt{2k}} W_{e^{2kt}-1}\right] \leq & \frac{\alpha_t - Floor}{\beta} \\ \Rightarrow e^{-kt} logP_0{}^Q + a(1-e^{-kt}) + e^{-kt}\frac{\sigma_1}{\sqrt{2k}} W_{e^{2kt}-1} \leq & log\left(\frac{\alpha_t - Floor}{\beta}\right) \end{aligned}$$

It follows that:

$$W_{e^{2kt}-1} \leq \frac{log(\frac{\alpha_t - Floor}{\beta}) - e^{-kt} logP_0{}^Q - a(1-e^{-kt})}{e^{-kt}\frac{\sigma_1}{\sqrt{2k}}}$$

We standardize the Brownian motion under the physical probability measure ($\mathbb{P}$):

$$\frac{W_{e^{2kt}-1}}{\sqrt{e^{2kt}-1}} \leq \frac{log(\frac{\alpha_t - Floor}{\beta}) - e^{-kt} logP_0{}^Q - a(1-e^{-kt})}{e^{-kt}\frac{\sigma_1}{\sqrt{2k}}\sqrt{e^{2kt}-1}}$$

Let us denote the RHS of the inequality by:

$$q_3 = \frac{log\left(\frac{\alpha_t - Floor}{\beta}\right) - e^{-kt} logP_0{}^Q - a(1-e^{-kt})}{e^{-kt}\frac{\sigma_1}{\sqrt{2k}}\sqrt{e^{2kt}-1}}$$

This brings us to the following result:

$$\mathbb{P}\left(P_t{}^Q \leq \frac{\alpha_t - Floor}{\beta}\right) = \mathbb{P}\left(\frac{W_{e^{2kt}-1}}{\sqrt{e^{2kt}-1}} \leq q_3\right) = \mathbb{N}(q_3) \tag{21}$$

On the other hand, we are interested in finding out $\mathbb{P}(P_t{}^Q \geq \frac{\alpha_t - I_0}{\beta})$:

$$\begin{aligned} P_t{}^Q \leq & \frac{\alpha_t - I_0}{\beta} \\ \Rightarrow exp\left[e^{-kt} logP_0{}^Q + a(1-e^{-kt}) + e^{-kt}\frac{\sigma_1}{\sqrt{2k}} W_{e^{2kt}-1}\right] \geq & \frac{\alpha_t - I_0}{\beta} \\ \Rightarrow e^{-kt} logP_0{}^Q + a(1-e^{-kt}) + e^{-kt}\frac{\sigma_1}{\sqrt{2k}} W_{e^{2kt}-1} \geq & log\left(\frac{\alpha_t - I_0}{\beta}\right) \end{aligned}$$

Or, equivalently:

$$W_{e^{2kt}-1} \geq \frac{log(\frac{\alpha_t - I_0}{\beta}) - e^{-kt} logP_0{}^Q - a(1-e^{-kt})}{e^{-kt}\frac{\sigma_1}{\sqrt{2k}}}$$

After standardizing, it follows that:

$$\frac{W_{e^{2kt}-1}}{\sqrt{e^{2kt}-1}} \geq \frac{log(\frac{\alpha_t - I_0}{\beta}) - e^{-kt} log P_0{}^Q - a(1-e^{-kt})}{\frac{\sigma_1}{\sqrt{2k}}\sqrt{1-e^{-2kt}}} = q_1$$

Then:

$$\begin{aligned}\mathbb{P}\left(P_t{}^Q \geq \frac{\alpha_t - I_0}{\beta}\right) =& \mathbb{P}\left(\frac{W_{e^{2kt}-1}}{\sqrt{e^{2kt}-1}} \geq q_1\right)\\ =& \mathbb{P}\left(\frac{W_{e^{2kt}-1}}{\sqrt{e^{2kt}-1}} \leq -q_1\right)\\ =& \mathbb{N}(-q_1)\end{aligned}$$

$$\mathbb{P}\left(P_t{}^Q \geq \frac{\alpha_t - I_0}{\beta}\right) = \mathbb{N}(-q_1) \tag{22}$$

We can now replace in the expression for T_1 with the computed probabilities (the results in 21 and 22):

$$T_1 = I_0 \cdot \{1 - [\mathbb{1}_{Floor \leq I_0} \cdot (1 - \mathbb{N}(q_3)) + \mathbb{N}(-q1) \cdot \mathbb{N}(q3)]\} \tag{23}$$

$$= I_0 \cdot [1 - \mathbb{1}_{Floor \leq I_0} \mathbb{N}(-q_3)) - \mathbb{N}(-q1)\mathbb{N}(q3)] \tag{24}$$

It is now time to focus on the computation of T_2:

$$\begin{aligned}T_2 =& \mathbb{E}_0[D_t \cdot \mathbb{1}_{D_t \leq I_0}]\\ =& \mathbb{E}_0\left[max(\alpha_t - \beta P_t{}^Q, Floor) \cdot \mathbb{1}_{D_t \leq I_0}\right]\\ =& \mathbb{E}_0\left[Floor \cdot \mathbb{1}_{D_t \leq I_0} \cdot \mathbb{1}_{\alpha_t - \beta P_t{}^Q < Floor}\right] + \mathbb{E}_0\left[(\alpha_t - \beta P_t{}^Q) \cdot \mathbb{1}_{D_t \leq I_0} \cdot \mathbb{1}_{\alpha_t - \beta P_t{}^Q \geq Floor}\right]\\ =& T_{21} + T_{22}\end{aligned}$$

Let us compute each expectation term separately:

$$\begin{aligned}T_{21} =& \mathbb{E}_0\left[Floor \cdot \mathbb{1}_{D_t \leq I_0} \cdot \mathbb{1}_{\alpha_t - \beta P_t{}^Q < Floor}\right]\\ =& Floor \cdot \mathbb{E}_0\left[\mathbb{1}_{max(\alpha_t - \beta P_t{}^Q, Floor) \leq I_0} \cdot \mathbb{1}_{\alpha_t - \beta P_t{}^Q < Floor}\right]\\ =& Floor \cdot \mathbb{E}_0\left[\mathbb{1}_{Floor \leq I_0} \cdot \mathbb{1}_{\alpha_t - \beta P_t{}^Q < Floor}\right]\\ =& Floor \cdot \mathbb{1}_{Floor \leq I_0} \cdot \mathbb{E}_0\left[\mathbb{1}_{\alpha_t - \beta P_t{}^Q < Floor}\right]\\ =& Floor \cdot \mathbb{1}_{Floor \leq I_0} \cdot \mathbb{P}\left(P_t{}^Q > \frac{\alpha_t - Floor}{\beta}\right)\end{aligned}$$

After replacing with the result in equation 22, it follows that:

$$T_{21} = Floor \cdot \mathbb{1}_{Floor \leq I_0} \cdot \mathbb{N}(-q_3) \tag{25}$$

$$\begin{aligned}
T_{22} =&\mathbb{E}_0\left[(\alpha_t - \beta P_t{}^Q)\cdot \mathbb{1}_{D_t\leq I_0}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\geq Floor}\right]\\
=&\alpha_t\cdot \mathbb{E}_0\left[\mathbb{1}_{D_t\leq I_0}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\geq Floor}\right] - \beta\cdot \mathbb{E}_0\left[P_t{}^Q\cdot \mathbb{1}_{D_t\leq I_0}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\geq Floor}\right]\\
=&T_{221}+T_{222}
\end{aligned}$$

We compute the two expectation terms separately:

$$\begin{aligned}
T_{221} =&\alpha_t\cdot \mathbb{E}_0\left[\mathbb{1}_{D_t\leq I_0}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\geq Floor}\right]\\
=&\alpha_t\cdot \mathbb{E}_0\left[\mathbb{1}_{max(\alpha_t-\beta P_t{}^Q,Floor)\leq I_0}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\geq Floor}\right]\\
=&\alpha_t\cdot \mathbb{E}_0\left[\mathbb{1}_{\alpha_t-\beta P_t{}^Q\leq I_0}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\geq Floor}\right]\\
=&\alpha_t\cdot \mathbb{E}_0\left[\mathbb{1}_{\alpha_t-\beta P_t{}^Q\leq I_0}\right]\cdot \mathbb{E}_0\left[\mathbb{1}_{\alpha_t-\beta P_t{}^Q\geq Floor}\right]\\
=&\alpha_t\cdot \mathbb{P}\left(P_t{}^Q\geq \frac{\alpha_t-I_0}{\beta}\right)\cdot \mathbb{P}\left(P_t{}^Q\leq \frac{\alpha_t-Floor}{\beta}\right)
\end{aligned}$$

$$\Rightarrow T_{221} = \alpha_t\cdot \mathbb{N}(-q_1)\cdot \mathbb{N}(q_3) \tag{26}$$

$$\begin{aligned}
T_{222} =&\beta\cdot \mathbb{E}_0\left[P_t{}^Q\cdot \mathbb{1}_{D_t\leq I_0}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\geq Floor}\right]\\
=&\beta\cdot \mathbb{E}_0\left[P_t{}^Q\cdot \mathbb{1}_{max(\alpha_t-\beta P_t{}^Q,Floor)\leq I_0}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\geq Floor}\right]\\
=&\beta\cdot \mathbb{E}_0\left[P_t{}^Q\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\leq I_0}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\geq Floor}\right]\\
=&\beta\cdot \mathbb{E}_0\left[e^{\left(e^{-kt}\log P_0{}^Q+a(1-e^{-kt})+e^{-kt}\frac{\sigma_1}{\sqrt{2k}}W_{e^{2kt}-1}\right)}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\leq I_0}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\geq Floor}\right]\\
=&\beta\cdot e^{\left(e^{-kt}\log P_0{}^Q+a(1-e^{-kt})\right)}\cdot \mathbb{E}_0\left[e^{\left(e^{-kt}\frac{\sigma_1}{\sqrt{2k}}W_{e^{2kt}-1}\right)}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\leq I_0}\cdot \mathbb{1}_{\alpha_t-\beta P_t{}^Q\geq Floor}\right]
\end{aligned}$$

We rely on Girsanov's Theorem in order to be able to solve the expectation. Let us define a new Brownian motion: $W_x{}^Q = W_x - \theta x$, with the Rhadon-Nykodim derivative: $\xi_t = \frac{d\mathbb{Q}}{d\mathbb{P}} = exp\left(-\frac{\theta^2}{2}x+\theta W_x\right)$. In our case $x = e^{2kt}-1$ and $\theta = e^{-kt}\frac{\sigma_1}{\sqrt{2k}}$. It follows that:

$$\begin{aligned}
\xi_t =&exp\left(-\frac{1}{2}e^{-2kt}\frac{\sigma_1{}^2}{2k}(e^{2kt}-1)+e^{-kt}\frac{\sigma_1}{\sqrt{2k}}W_{e^{2kt}-1}\right)\\
=&exp\left(\frac{\sigma_1{}^2}{4k}(e^{-2kt}-1)+e^{-kt}\frac{\sigma_1}{\sqrt{2k}}W_{e^{2kt}-1}\right)
\end{aligned}$$

We can now apply the rule: $\mathbb{E}^{\mathbb{P}}[a\cdot\xi_t] = \mathbb{E}^{\mathbb{Q}}[a]$. Then:

$$
\begin{aligned}
T_{222} =&\beta \cdot e^{\left(e^{-kt}\log P_0{}^Q + a(1-e^{-kt})\right)} \cdot \mathbb{E}_0 \left[e^{\left(-\frac{\sigma_1{}^2}{4k}(e^{-2kt}-1)\right)} \cdot e^{\left(\frac{\sigma_1{}^2}{4k}(e^{-2kt}-1)+e^{-kt}\frac{\sigma_1}{\sqrt{2k}}W_{e^{2kt}-1}\right)} \cdot \mathbb{1}_{\alpha_t - \beta P_t{}^Q \le I_0} \cdot \mathbb{1}_{\alpha_t - \beta P_t{}^Q \ge Floor} \right] \\
=&\beta \cdot e^{\left(e^{-kt}\log P_0{}^Q + a(1-e^{-kt}) + \frac{\sigma_1{}^2}{4k}(1-e^{-2kt})\right)} \cdot \mathbb{E}_0 \left[\mathbb{1}_{\alpha_t - \beta P_t{}^Q \le I_0} \cdot \mathbb{1}_{\alpha_t - \beta P_t{}^Q \ge Floor} \right] \\
=&\beta \cdot e^{\left(e^{-kt}\log P_0{}^Q + a(1-e^{-kt}) + \frac{\sigma_1{}^2}{4k}(1-e^{-2kt})\right)} \cdot \mathbb{Q}\left(P_t{}^Q \ge \frac{\alpha_t - I_0}{\beta} \right) \cdot \mathbb{Q}\left(P_t{}^Q \le \frac{\alpha_t - Floor}{\beta} \right)
\end{aligned}
$$

Let us now compute the two probabilities under the new probability measure. For the first probability we have that:

$$
P_t{}^Q \ge \frac{\alpha_t - I_0}{\beta} \Leftrightarrow W_{e^{2kt}-1} \ge \frac{log(\frac{\alpha_t - I_0}{\beta}) - e^{-kt} log P_0{}^Q - a(1-e^{-kt})}{e^{-kt}\frac{\sigma_1}{\sqrt{2k}}}
$$

But $W_{e^{2kt}-1}$ is no longer a standard Brownian motion under $\mathbb{Q}$. We need to standardize it, by bringing its mean to zero:

$$
\frac{W_{e^{2kt}-1} - e^{-kt}(e^{2kt}-1)\frac{\sigma_1}{\sqrt{2k}}}{\sqrt{e^{2kt}-1}} \ge \frac{log(\frac{\alpha_t - I_0}{\beta}) - e^{-kt} log P_0{}^Q - a(1-e^{-kt})}{e^{-kt}\frac{\sigma_1}{\sqrt{2k}}\sqrt{e^{2kt}-1}} - \frac{e^{-kt}\frac{\sigma_1}{\sqrt{2k}}(e^{2kt}-1)}{\sqrt{e^{2kt}-1}}
$$

$$
\Rightarrow \frac{W_{e^{2kt}-1} - e^{-kt}(e^{2kt}-1)\frac{\sigma_1}{\sqrt{2k}}}{\sqrt{e^{2kt}-1}} \ge q_1 - e^{-kt}\frac{\sigma_1}{\sqrt{2k}}\sqrt{e^{2kt}-1} = q_2
$$

$$
\begin{aligned}
\mathbb{Q}\left(P_t{}^Q \ge \frac{\alpha_t - I_0}{\beta} \right) =&\mathbb{Q}\left(\frac{W_{e^{2kt}-1} - e^{-kt}(e^{2kt}-1)\frac{\sigma_1}{\sqrt{2k}}}{\sqrt{e^{2kt}-1}} \ge q_2 \right) \\
=&\mathbb{Q}\left(\frac{W_{e^{2kt}-1} - e^{-kt}(e^{2kt}-1)\frac{\sigma_1}{\sqrt{2k}}}{\sqrt{e^{2kt}-1}} \le -q_2 \right) \\
=&N(-q_2)
\end{aligned}
$$

For computing the second probability $\mathbb{Q}(P_t{}^Q \le \frac{\alpha_t - Floor}{\beta})$ we employ a similar approach:

$$
P_t{}^Q \le \frac{\alpha_t - Floor}{\beta} \Leftrightarrow W_{e^{2kt}-1} \le \frac{log(\frac{\alpha_t - Floor}{\beta}) - e^{-kt} log P_0{}^Q - a(1-e^{-kt})}{e^{-kt}\frac{\sigma_1}{\sqrt{2k}}\sqrt{e^{2kt}-1}}
$$

After standardizing it:

$$
\frac{W_{e^{2kt}-1} - e^{-kt}(e^{2kt}-1)\frac{\sigma_1}{\sqrt{2k}}}{\sqrt{e^{2kt}-1}} \le q_3 - \frac{\sigma_1}{\sqrt{2k}}\sqrt{1-e^{-2kt}} = q_4
$$

It follows that:

$$\mathbb{Q}\left({P_t}^Q \leq \frac{\alpha_t - Floor}{\beta}\right) = \mathbb{N}(q_4)$$

Finally, by replacing with the expressions obtained for each probability, we can write the entire expression for T_{222}:

$$T_{222} = \beta \cdot e^{\left(e^{-kt}\log {P_0}^Q + a(1-e^{-kt}) + \frac{{\sigma_1}^2}{4k}(1-e^{-2kt})\right)} \cdot \mathbb{N}(-q_2) \cdot \mathbb{N}(q4) \tag{27}$$

Given that $T_{22} = T_{221} - T_{222}$ and replacing with the expressions obtained in 26 and 27, we have that:

$$T_{22} = \alpha_t \cdot \mathbb{N}(-q_1) \cdot \mathbb{N}(q_3) - \beta \cdot e^{\left(e^{-kt}\log {P_0}^Q + a(1-e^{-kt}) + \frac{{\sigma_1}^2}{4k}(1-e^{-2kt})\right)} \cdot \mathbb{N}(-q_2) \cdot \mathbb{N}(q4) \tag{28}$$

Also, we know that $T_2 = T_{21} + T_{22}$ and we rely on the results 25 and 28:

$$T_2 = Floor \cdot \mathbb{1}_{Floor \leq I_0} \cdot \mathbb{N}(-q_3) + \alpha_t \cdot \mathbb{N}(-q_1) \cdot \mathbb{N}(q_3) - \tag{29}$$

$$\beta \cdot e^{\left(e^{-kt}\log {P_0}^Q + a(1-e^{-kt}) + \frac{{\sigma_1}^2}{4k}(1-e^{-2kt})\right)} \cdot \mathbb{N}(-q_2) \cdot \mathbb{N}(q4) \tag{30}$$

As a last step, we can reconstruct $\mathbb{E}_0[min(I_0, D_t)] = T_1 + T_2$ with the help of equations 23 and 29:

$$\begin{aligned}\mathbb{E}_0[min(I_0, D_t)] = I_0 \cdot [1 - \mathbb{1}_{Floor \leq I_0}\mathbb{N}(-q_3)) - \mathbb{N}(-q1)\mathbb{N}(q3)] + Floor \cdot \mathbb{1}_{Floor \leq I_0} \cdot \mathbb{N}(-q_3) \\ + \alpha_t \mathbb{N}(-q_1) \cdot \mathbb{N}(q_3) - \beta e^{e^{-kt}\log {P_0}^Q + a(1-e^{-kt}) + \frac{{\sigma_1}^2}{4k}(1-e^{-2kt})}\mathbb{N}(-q_2)\mathbb{N}(q4)\end{aligned}$$

B Model Specifications - Simulation of Permit and Output Prices and Output Demand

Monte Carlo Simulation of Carbon Prices

We allow the manager to trade carbon allowances only once at the end of each month. We assume carbon prices move according to a GBM with jumps, as described above. In between manager's monthly trades, carbon prices are allowed to evolve daily, for an average of twenty trading days per month. In order to simulate the price process, we discretize the solution to the price equation:

$$P_t^{CO_2} = P_{t-\Delta t}^{CO_2} \cdot e^{\mu_2 \Delta t} \cdot e^{(-\frac{\sigma_2^2}{2}\Delta t + \sigma_2 \sqrt{\Delta t}\varepsilon_t)} \cdot e^{[(N_t - N_{t-\Delta t})\log(1+\phi)\sqrt{\Delta t} - \lambda\phi\sqrt{\Delta t}]} \tag{31}$$

where $\varepsilon_t \sim \mathbb{N}(0,1)$, and $\Delta t = \frac{1}{20}$. In order to simulate the jumps, we first model the jump times τ_1, τ_2, etc. explicitly. Each jump time is extracted independently from the exponential distribution, i.e. $\tau \sim$ Exponential($\frac{1}{\lambda}$). From one jump to the next, the carbon price evolves like an ordinary GBM, under the assumption that the Brownian Motion and the Jump process move independently from each other.

Monte Carlo Simulation of Commodity Prices

In a similar way to allowance prices, we allow the output price to evolve daily, this time following a mean-reverting process, as described in Section 3.2. Again, the firm sells output only once at the end of each month, at the then prevailing price. We allow for positive correlation between carbon allowances and commodity prices, and for simulating the output price process, we decompose the Geometric Brownian Motion W_t into two independent processes. We are therefore able to discretize the output price:

$$P_t^Q = e^{e^{-k\Delta t}\log P_{t-\Delta t}^Q + a(1-e^{-k\Delta t}) + e^{-k\Delta t}\frac{\sigma_1}{\sqrt{2k}}(e^{2k\Delta t}-1)(\rho\varepsilon_t + \sqrt{1-\rho^2}\xi_t)} \tag{32}$$

where ε_t is the standard normal movement in the CO_2 price, while the $\xi_t \sim \mathbb{N}(0,1)$ is an independent random variable expressing the uncorrelated movement in output price.

Calibration Parameters

Our solution approach to the manager's problem, as formulated in Section 3.1, is numerical and requires, therefore, the definition of various parameters. We consider a *standard* set of parameters in order to solve the numerical example.

Table IV: **Model Calibration Parameters - Carbon and Commodity Prices and Demand**

Variable	Parameter	Value	Denomination
Demand	D_0	20	units
	β	0.1	
	g	0.2	
	Floor	5	
Output Price	${P_t}^Q$	20	$EUR/unit$
	μ_1	$log({P_t}^Q)$	
	σ_1	0.05	
	k	0.4	
$corr(B_t, W_t)$	ρ	0.2	$EUR/unit$
CO_2 Price	${P_0}^{CO_2}$	5	EUR
	μ_2	0.1	
	σ_2	0.1	
	λ	0.02	
	ϕ	-0.2	

Note: The table illustrates the parameters used for the simulation of the stochastic processes of output and CO_2 permit prices and output demand. Graphical illustrations of possible paths using these parameters are presented in Figures 4, 5, and 6 below. This is the *standard* set of parameters used for the main results in Section 4.2. For all processes, we have chosen moderate parameter values, trying to avoid peripheral cases that might increase the chance of extreme (corner) solutions. First of all, the initial levels of prices and demand $(D_0, {P_t}^Q, {P_0}^{CO_2})$ have been chosen arbitrarily, but paying attention that profit levels make economic sense. We have chosen the model and our parameters using their estimations of Daskalakis et al. [2009] as guiding case, keeping an eye on the magnitude and adjusting them to match less extreme cases. The modeling of the output demand process was inspired by Li and Gu [2012] and we have used their parameter values. For the output price parameters, the paper of Schwartz [1997] was of great help. We relaxed these assumptions during the sensitivity analysis, and allowed for different parameter values in the quest of seeing the influence on the final results.

Below, we display various possible paths for the output and permit prices and output demand over a period of one year (250 trading days). The simulations were realized based on the parameter values described in Table IV and the dynamics presented in Section 3.2.

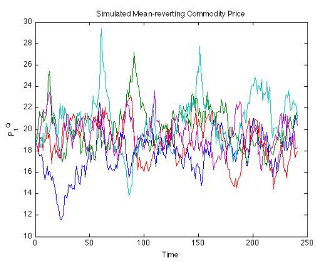

Figure 4: Output Price

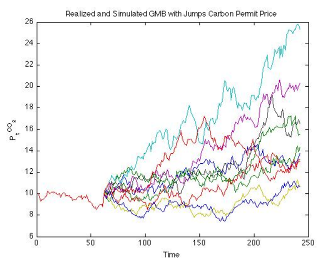

Figure 5: Carbon Permit Price

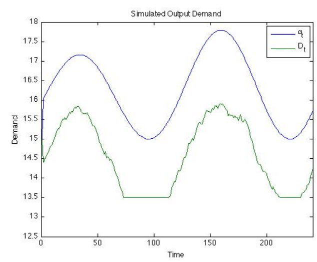

Figure 6: Output Demand

Table V: **Model Calibration Parameters - Additional Inputs**

Parameter	Symbol	Value	Denomination
Periods	n	16	quarters
Endowment	N	20	permits
Conversion Factor	Ω	1	$tCO_2/unit$
Production Cost	a_1	0.08	$EUR/unit$
Production Cost	a_2	0.5	$EUR/unit$
Storage Cost	h	1	$EUR/unit$
Interest Rate	r	10%	yearly
Penalty	Pen	100	EUR/tCO_2
Pr Bankruptcy	ψ_S	0.05	

Note: The table captures the additional inputs needed for running our model. We rely on moderate value levels for the parameters. The *Conversion Factor* helps with performing the equivalence between units of output produced and tons of $CO_{2,e}$ emitted. The two parameters of the *Production Cost* (a_1, a_2) are relevant for the computation of the quadratic cost function ($C(y_t) = a_1 y_t + a_2 {y_t}^2$). The *Penalty level* corresponds to the regulations of the second phase of EU ETS.

Table VI: **Model Calibration Parameters - Selection of solution intervals**

Parameter	Symbol	Lower Bound	Upper Bound
Sales	x_t	0	$\mathbb{E}_{t-1}[min(D_t, I_{t-1})]$
Production	y_t	0	30
Permit trade	z_t	$-\left(\sum_{1}^{t-1} x_i + N\right)$	N
Inventory	I_t	0	20

C Introducing Uncertainty Regarding the Number of Permits Available for Trading

- The possible permit transactions are limited for the agent from both the upside and the downside. From the upside, the maximum number of permits the agent could buy is bounded by:

$$maxLong = LongLimit * N \tag{33}$$

  where N is the initial endowment of permits.
  The sales of permits are bounded at each moment of time by the number of permits actually held, i.e. the agent cannot go short on the permits' market:

$$maxShort_t = \sum_{i=0}^{t-1} z_i + N \tag{34}$$

  This directly implies that, given unlimited demand and supply, the number of permits the agent can sell or buy at time t will be in the range: $z_t \in [maxShort; maxLong]$.

- We assume uncertainty comes in the form of the available demand/supply of carbon permits on the market.

- We define the random supply of permits on the market for our agent as a percentage of his maximum long position (maxLong):

$$Supply_t = {\omega_t}^L * maxLong_t \tag{35}$$

  The random demand of permits is similarly defined as a percentage of the maximum short position (maxShort):

$$Demand_t = {\omega_t}^S * maxShort_t \tag{36}$$

- At each moment of time, the proportion of available permits on the market (ω_t) is random. The agent is aware of the current supply/demand of permits, but with regard to future quantities of permits available, he knows only the possible outcomes and the probability associated with them.

- We assume the proportion of permits available on the market at each moment of time follows a uniform discrete distribution. The distribution is assumed to be stationary over time. Possible outcomes are detailed in the table below:

Table VII: **Proportion of Available Permits - Possible outcomes**

ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	$\mathbb{E}[\omega]$
0.4	0.5	0.6	0.7	0.8	0.9	1	0.7

- At each moment of time, the agent receives information regarding the available quatities he could buy or sell. The proportions of permits available are different and independent for long and short positions. For example, at one particular moment, out of the total amount the agent could sell there could be a demand for only 80% of these permits. At the same time, for the maximum amount of permits that the agent could buy there could be a total supply for only 70% of these permits.

- Lower quantities of permits to buy or sell than the maximum held by the agent, have an associated higher probability of being available, depending how far are these quantities from the maximum quantity. These proportions are calculated according to the following formula:

$$\omega_i^S = min(1,(\omega_S + (indexS/nS)/10)); \quad (37)$$
$$\omega_i^L = min(1,(\omega_L + (indexL/nL)/10)); \quad (38)$$

  where nS and nL represent the total number of considered short, respectively long positions. IndexS and IndexL refer to the position of the amount of permits compared to the maximum short/long quantity. The following table helps explicit these calculations:

Table VIII: **Possible Permit Trading and Availabilities**

z_t	Index	Proportion
-60	1	0.70
-50	2	0.72
-40	3	0.73
-30	4	0.75
-20	5	0.77
-10	6	0.78
0	1	1
10	3	0.77
20	2	0.73
30	1	0.70

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